Numerical Simulations of Turbulent Mixing in Eruption Clouds

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Eruption clouds in explosive volcanic eruptions are a kind of free boundary shear flow with very high Reynolds numbers ($Re > 10^8$), and their dynamics are governed by entrainment of ambient air into eruption clouds by turbulent mixing. We developed a numerical pseudo-gas model of an eruption cloud by employing three-dimensional coordinates, a high-order accuracy scheme, and a fine grid size in order to investigate the behavior of entrainment due to turbulent mixing. Our results have revealed that the efficiency of turbulent mixing no longer depends on the Reynolds number when the turbulence is fully developed. Our model has successfully reproduced the features of turbulent mixing at high Reynolds numbers as well as fundamental features of the dynamics of eruption clouds, such as the generation of eruption columns and/or pyroclastic flows.

Keywords: volcanic eruption cloud, pseudo-gas model, turbulent mixing, volcanic hazard

1. Introduction

During explosive volcanic eruptions, a mixture of hot ash (solid pyroclasts) and volcanic gas is released from the volcanic vent into the atmosphere. Such events are characterized by the formation of eruption columns and/or pyroclastic flows. Turbulent mixing in and around eruption clouds is an essential part of the dynamics of eruption clouds because the amount of entrained air controls whether or not the eruption cloud becomes buoyant [1].

Over the past 20 years, several two-dimensional (2-D), time-dependent numerical models for eruption clouds have been developed in order to explain fluid dynamical features of explosive volcanism [2, 3]. These studies focused on the unsteady and multiphase features of eruption clouds. However, the features of turbulent mixing reproduced by their models were not quantitatively consistent with those observed in the laboratory experiments [4]. Here, we develop a three-dimensional (3-D) numerical model, which correctly reproduces the quantitative features of turbulent mixing in and around the turbulent jet. Our new model is applicable to time-dependent phenomena in actual volcanological situations.

2. Model Description

The numerical model of eruption cloud is based on the model of Suzuki et al. [4]. The model is designed to describe the injection of a mixture of solid pyroclasts and volcanic gas from a circular vent above a flat surface of the earth in a stationary atmosphere. In this study, because we are particularly concerned with turbulent mixing of eruption clouds, we adopt a pseudo-gas model; we ignore the separation of solid pyroclasts from the eruption cloud. The fluid dynamics model solves a set of partial differential equations describing the conservation of mass, momentum, and energy, and a set of constitutive equations describing the thermodynamic state of the mixture of solid pyroclasts, volcanic gas, and air. These equations are solved numerically by the Roe scheme [5]. The MUSCL method is applied to interpolate the fluxes between grid points [6]. The calculations are performed on axisymmetric 2-D and 3-D domains.

One of the most essential physics which governs the dynamics of eruption clouds is that the density of eruption clouds varies nonlinearly with the mixing ratio between the ejected material and air. Generally, the ejected material has an initial density of several times as large as the atmospheric density since it contains more than 90 wt% solid pyroclasts at the vent [7]. As the ejected material is mixed with ambient air and the mass fraction of the ejected material decreases, the density of the mixture drastically decreases and becomes less than the atmospheric density because the entrained air expands by heating from the hot solid pyroclasts.

We reproduce this nonlinear feature of mixture density by changing the effective gas constant and the effective specific heat of the mixture as

$$R_m = n_a R_a + n_g R_g, \tag{1}$$

$$C_{vm} = n_a C_{va} + n_g C_{vg} + n_s C_{vs}, \tag{2}$$

where R is the gas constant, n is the mass fraction, and C_v is the specific heat at constant volume. The subscripts m, a, g, and s refer to the mixture, air, volcanic gas, and solid pyroclasts, respectively. The mass fractions of air (n_a), volcanic gas (n_g), and solid pyroclasts (n_s) satisfy the condition of $n_a + n_g + n_s = 1$. Using these procedures, the equation of state for the mixture of the ejected material and air can be approximated by the equation of state for an ideal gas

$$p = \rho R_m T, \quad (3)$$

where p is the pressure, ρ is the density of the mixture, and T is the temperature.

3. Turbulent Mixing

Generally speaking, the efficiency of turbulent mixing is a function of the Reynolds number [8]. At $Re < 10^4$, even though the flow may be unsteady, the resulting turbulent flow cannot be described as fully developed and the efficiency of mixing increases with Re. On the other hand, at $Re > 10^4$ the turbulence is fully developed, and the large-scale structures of the engulfment process and the efficiency of entrainment no longer depend on Re. Because the flow of eruption clouds is considered to be fully turbulent, the simulations of eruption clouds should be carried out under the condition where the efficiency of mixing no longer depends on the numerical Reynolds number.

3.1 Turbulent Jets in Uniform Environments

In the case of turbulent jet which is ejected from a nozzle into a uniform environment, the flow is characterized by the self-similarity that the radial length scale is proportional to the distance from the nozzle. Experimental studies have revealed that the horizontal profile of jet can be approximated by the Gaussian profile and its width represents the efficiency of entrainment.

We simulate the turbulent jet with variable numerical conditions and investigate the effects of three-dimensionality and spatial resolution on the quantitative features of turbulent mixing in the numerical simulations. In the 3-D simulations, the radius of jet increases linearly with height (i.e., self-similarity) (Fig. 1a). On the other hand, in the 2-D simulations, the ejected fluid rises along the central axis and the spreading rate of the jet is substantially smaller than those of the 3-D simulations (Fig. 1b). The simulations of the high-order accuracy scheme with a fine grid size reproduce the turbulence containing the various scales of vortices and the self-similarity (Fig. 1a). On the other hand, in the simulations of the first-order accuracy scheme with the same grid size (Fig. 1c) or the high-order accuracy scheme with a coarse grid size (Fig. 1d), the vortical structures of the jet are not correctly reproduced.

We systematically examined the effects of grid size on efficiency of turbulent mixing in the numerical simulations. We found that the width of turbulent jet is independent of the grid size when the grid size is small (curves a and e in Fig. 2). On the other hand, in the simulations of the 2-D coordinates (curve b) or the low spatial resolutions (curves c and d), the width of jet is substantially small. Fig. 3 shows that the

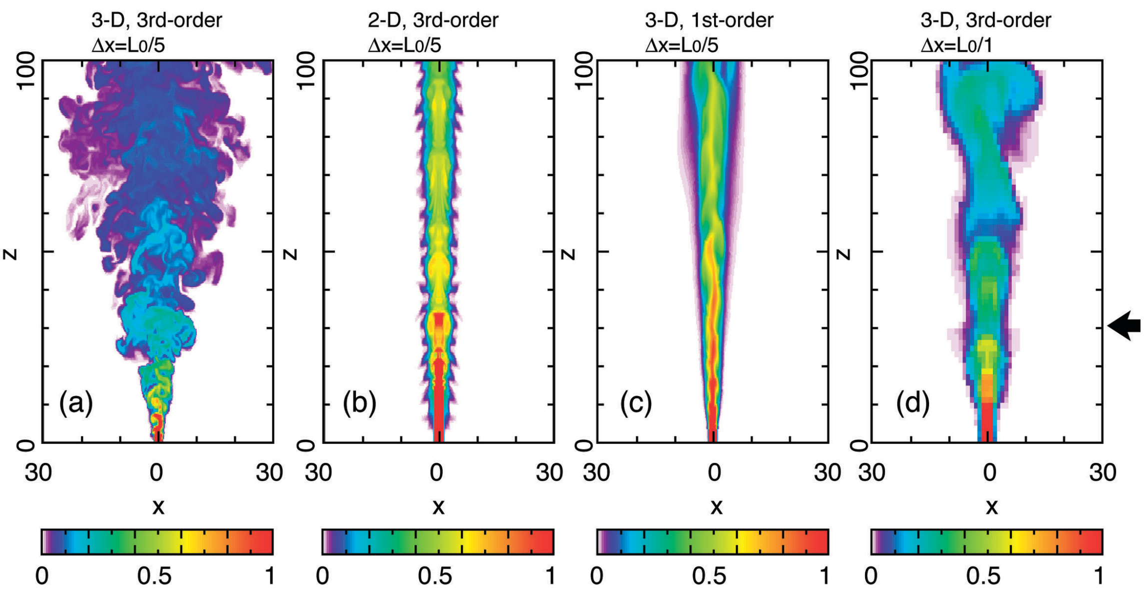

Fig. 1 Numerical results of turbulent jets ejected into the same fluid. The color illustrates the cross-sectional distribution of the mass fraction of the ejected fluid. The horizontal distance from the centerline and the vertical distance from the nozzle are represented by x and z, respectively. (a) Simulation of the third-order accuracy scheme with $\Delta x = L_0/5$ in 3-D coordinates where Δx is the grid size, L_0 is the nozzle radius. (b) Simulation of the third-order accuracy scheme with $\Delta x = L_0/5$ in 2-D coordinates. (c) Simulation of the first-order accuracy scheme with $\Delta x = L_0/5$ in 3-D coordinates. (d) Simulation of the third-order accuracy scheme with $\Delta x = L_0/1$ in 3-D coordinates.

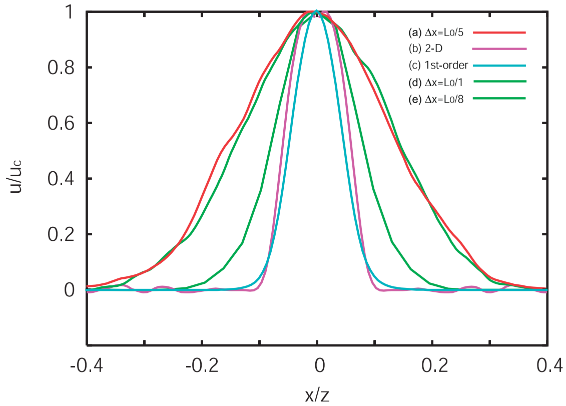

Fig. 2 Velocity profiles across a turbulent jet. Vertical axis represents the vertical velocity normalized by the centerline value (u_c). Horizontal axis represents dimensionless displacement (x/z). Curves a, b, c, and d are the time-averaged velocity profiles at fixed cross sections for the simulations of Fig. 1a, 1b, 1c, and 1d, respectively. The heights of the cross sections are shown by arrows in Fig. 1 (z = 30 m). Curve e illustrates the simulation of the third-order accuracy scheme in 3-D coordinates with $\Delta x = L_0/8$.

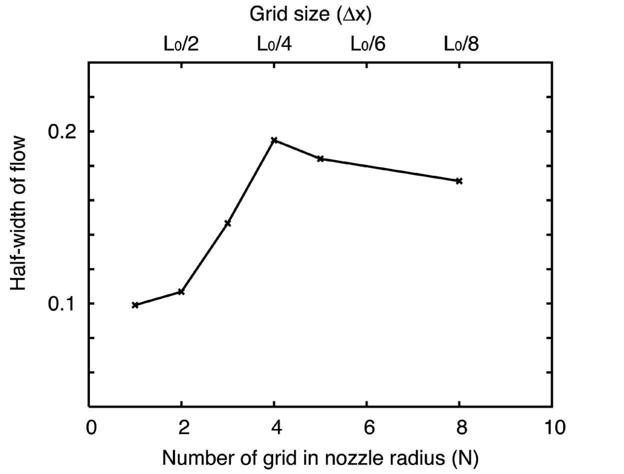

Fig. 3 Half-width of flow as a function of the number of grid points in nozzle radius when the vertical profile is approximated by the Gaussian profile. The heights of the cross sections are shown by arrows in Fig. 1 (z = 30 m).

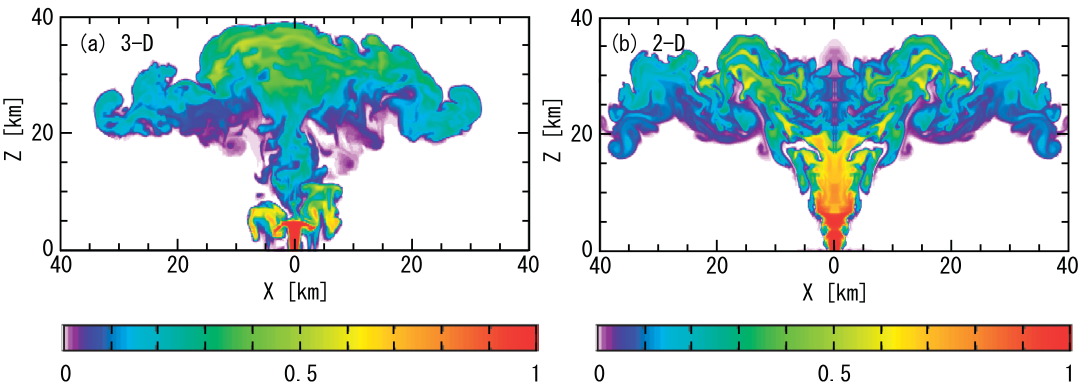

Fig. 4 Numerical results of eruption clouds at 500 s from the beginning of eruption. The color illustrates the cross-sectional distribution of the mass fraction of the ejected material (volcanic gas plus solid pyroclasts). An initial temperature of the mixture is 1000 K, an initial mass fraction of the volcanic gas is 0.05, the pressure at the vent is 1 atm, and the mass discharge rate is 10^9 kg/s. (a) Simulation of 3-D coordinates. (b) Simulation of axisymmetric 2-D coordinates.

width of flow increases as the number of grid (N) increases when N is less than 4, and that the width is independent of N when N is more than 4. In Fig. 3, the vertical axis (the half-width of jet) represents the efficiency of turbulent mixing and the horizontal axis (the number of grid in nozzle radius) is related to the numerical Reynolds number because the numerical Reynolds number increases by means of increasing spatial resolution. These results indicate that the condition for fully turbulence is considered to be achieved when N is more than 4. The above results are explained by the fact that the efficiency of entrainment is determined by the kinematic evolution of the largest eddies [e.g., 8].

3.2 Turbulent Mixing in Eruption Clouds

We simulate the eruption cloud with axisymmetric 2-D and 3-D coordinates and investigate the effects of three-dimensionality on the dynamics of eruption clouds. In the simulations, an initial temperature of the mixture is 1000 K, an initial mass fraction of the volcanic gas is 0.05, the pressure at the vent is 1 atm, and the mass discharge rate is 10^9 kg/s.

Simulations reproduce the pyroclastic flows, the buoyant plume from the pyroclastic flows, and umbrella cloud. In the 3-D simulation, the axis of the flow fluctuates with height, which causes efficient turbulent mixing of the ejected and surrounding fluids (Fig. 4a). On the other hand, in the axisymmetric 2-D simulation, the buoyant plume rises along the central axis and the efficiency of turbulent mixing is significantly reduced because of the fixed boundary condition at the centerline of the axisymmetric coordinates (Fig. 4b).

4. Summary

In order to correctly reproduce the turbulent mixing process in eruption clouds, it is essential to apply 3-D coordinates with a sufficiently high spatial resolution. We should carefully perform sensitivity tests with different grid sized to find the condition where the efficiency of mixing no longer depends on the numerical Reynolds number (i.e., independent of grid sizes).

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噴煙内の乱流混合の数値シミュレーション

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本プロジェクトでは、大規模数値シミュレーションによって固体地球と地球表層・大気にまたがる火山現象の理解を目指している。特に、防災上重大な問題となる火山噴煙の拡大を定量的に再現・予測する数値モデルの開発に取り組んでいる。火山噴煙のダイナミクスにおいては、(1)噴出物(火山灰＋火山ガス)と大気からなる混合物の密度変化、(2)乱流による混合過程、が重要な役割を果たす。一般に、火口からの噴出物は固体である火山灰を90wt%以上含むため、周囲の大気よりも高密度である。しかし、周囲の大気が取り込まれると、取り込まれた大気が火山灰の熱によって急激に膨張し、噴出物と大気の混合物が周囲の大気よりも低密度になる。我々は、混合比によって理想気体の状態方程式における気体定数を変化させることによってこのような変則的な密度変化を再現する計算手法(擬似ガスモデル)を開発した。さらに、数値計算の大規模化によって噴煙柱や火砕流、傘型噴煙といった爆発的火山噴火に特徴的な噴煙現象を再現した。噴煙のような自由噴流の乱流混合に関しては、乱流が十分に発達した高レイノルズ数条件下で、その混合効率がレイノルズ数に依存しなくなることがこれまでの室内実験によって知られている。数値計算上、乱流混合の効率を定量的に再現するためには、この高レイノルズ数領域における乱流混合の漸近的性質を正しく再現する必要がある。具体的には、空間分解能など数値レイノルズ数を反映する計算条件を変えたときに、流れのグローバルな性質に関する計算結果が変化しないことが求められる。そこで、本プロジェクトの数値モデルについて流れのグローバルな性質に対する空間分解能や3次元性の影響を系統的に調べた結果、3次元座標系で高精度スキームを適用し、細かなグリッドサイズを用いることによって、乱流ジェットや火山噴煙のような高レイノルズ数自由噴流の乱流混合を定量的に再現できることが分かった。

キーワード：火山噴煙, 擬似ガスモデル, 乱流混合, 火山災害