Numerical Simulations of Turbulent Flow in Volcanic Eruption Clouds

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Explosive volcanism is one of the most catastrophic phenomena on the earth. The dynamics of eruption clouds in explosive volcanic eruptions are governed by turbulent flow; the amount of entrained air due to turbulent mixing affects total height of eruption column and the intensity of turbulence controls tephra dispersion in the atmosphere and resultant tephra-fall deposits. In order to investigate features of turbulent flow in the eruption cloud and its effect on the tephra dispersion, we are developing a numerical model for eruption cloud dynamics. The present 3-D model has successfully reproduced the qualitative and quantitative features of the eruption clouds with mass discharge rate ranging from 10^6 to 10^{10} kg/s. The 3-D simulations reveal that the efficiency of entrainment varies with height; the value of entrainment coefficient near the vent is smaller than that in the buoyant region. It is also suggested that the intensity of turbulence is not strong enough to homogenize particles inside umbrella clouds; particles coarser than a few mm tend to concentrate at the base of the umbrella clouds.

Keywords: volcanic eruption cloud, pseudo-gas model, turbulent mixing, volcanic hazard

1. Introduction

During an explosive volcanic eruption, hot volcanic gases and pyroclasts are ejected from a volcanic vent into the atmosphere, and the ejected materials mix with air and buoyantly rise as an eruption column. After the eruption column reaches the neutral buoyancy level, it flows horizontally to form an umbrella cloud [1].

Turbulent mixing in and around eruption columns is an essential part of the dynamics of eruption columns because the amount of entrained air controls whether or not the eruption cloud becomes buoyant. When the ejected material entrains sufficient air to become buoyant, a large plinian eruption column rises as a turbulent plume. If the ejected material does not entrain sufficient air and its vertical velocity falls to zero before the eruption cloud becomes buoyant, a column collapse occurs and the heavy cloud spreads as a pyroclastic flow. The dimensions of the eruption column such as the total height of the eruption column are controlled by the amount of the entrained air integrated from the vent to the top of the column. In umbrella clouds, turbulence also governs the tephra dispersion and resultant tephra-fall deposits.

The aim of this study is to develop a 3-D numerical model which can reproduce the behavior of eruption column and umbrella cloud and to investigate the features of turbulent mixing in eruption column (Section 3) and the effects of the intensity of turbulence in umbrella clouds on tephra dispersion (Section 4).

2. Model Description

The numerical model of eruption cloud is based on Suzuki et al. [2]. The model is designed to describe the injection of a mixture of solid pyroclasts and volcanic gas from a circular vent above a flat surface of the earth in a stationary atmosphere. We apply a pseudo-gas model; we ignore the separation of solid pyroclasts from the eruption cloud and treat an eruption cloud as a single gas. The fluid dynamics model solves a set of partial differential equations describing the conservation of mass, momentum, and energy, and a set of constitutive equations describing the thermodynamic state of the mixture of solid pyroclasts, volcanic gas, and air.

One of the most essential physics which governs the dynamics of eruption clouds is that the density of eruption clouds varies nonlinearly with the mixing ratio between the ejected material and air. We reproduce this nonlinear feature of mixture density by changing the effective gas constant and the effective specific heat of the mixture. Then, the equation of state for the mixture of the ejected material and air is approximated by the equation of state for an ideal gas. Because the above assumption of the equation of state makes it possible to derive analytically the eigenvalues and eigenvectors for the governing equations of the mixture, the Roe scheme [3] can be applied to the present problem of the dynamics of eruption clouds. The MUSCL method [4] is applied to interpolate the fluxes between grid points.

The calculations are performed on 3-D domain. We use a uniform grid in a Cartesian coordinates system. We have also developed a new 3-D code in which the domain is discretized on a non-uniform grid; the grid size is set to be $L_0/10$ near the vent, where L_0 is the vent radius, and it increases at a constant rate (equal to 1.02) with the distance from the vent up to 100 m.

3. Efficiency of Turbulent Mixing in Eruption Column

Eruption cloud is driven by the initial momentum as a turbulent jet near the vent and by buoyancy as a turbulent plume where the ambient air is sufficiently entrained. Generally speaking, the efficiency of turbulent mixing in the turbulent jet and plume can be measured by a proportionality constant relating the inflow velocity at the edge of the flow to the mean vertical velocity (i.e., entrainment coefficient, *k*) [5]. In the case of a pure jet and plume, the value of *k* is almost constant (~ 0.1). On the other hand, constant *k* is not always valid for the flow of eruption column because of the nonlinear feature of the equation of state and the stratification of atmosphere. Therefore, we attempt to estimate the value of *k* in the eruption column as a function of the downstream distance from the volcanic vent (referred to as 'local *k*').

We carried out the 3-D numerical simulations in the relatively small eruption with the mass discharge rate of $10^{6\text{–}7}$ kg/s. The present simulations have successfully reproduced an eruption column and the structures of turbulence (Fig. 1).

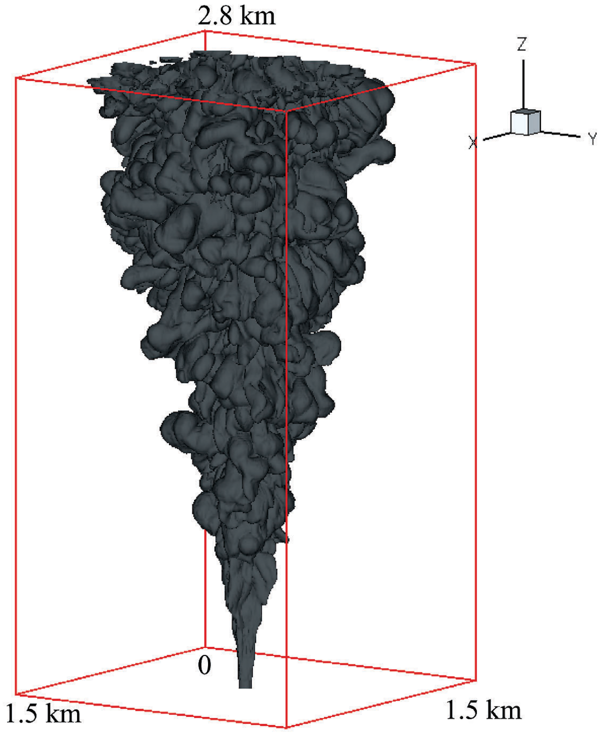

Fig. 1 A representative numerical result with the mass discharge rate of 10^6 kg/s. The iso-surface of the mass fraction of the magma of 10^{-3} is represented.

We have estimated the value of local *k* on the basis of the snapshots of these results (Fig. 2). First, we obtain the time-average distributions of physical quantities (Fig. 2b). Secondly, horizontally integrating the physical quantities, we measure the local values of the mean fluxes and charac-

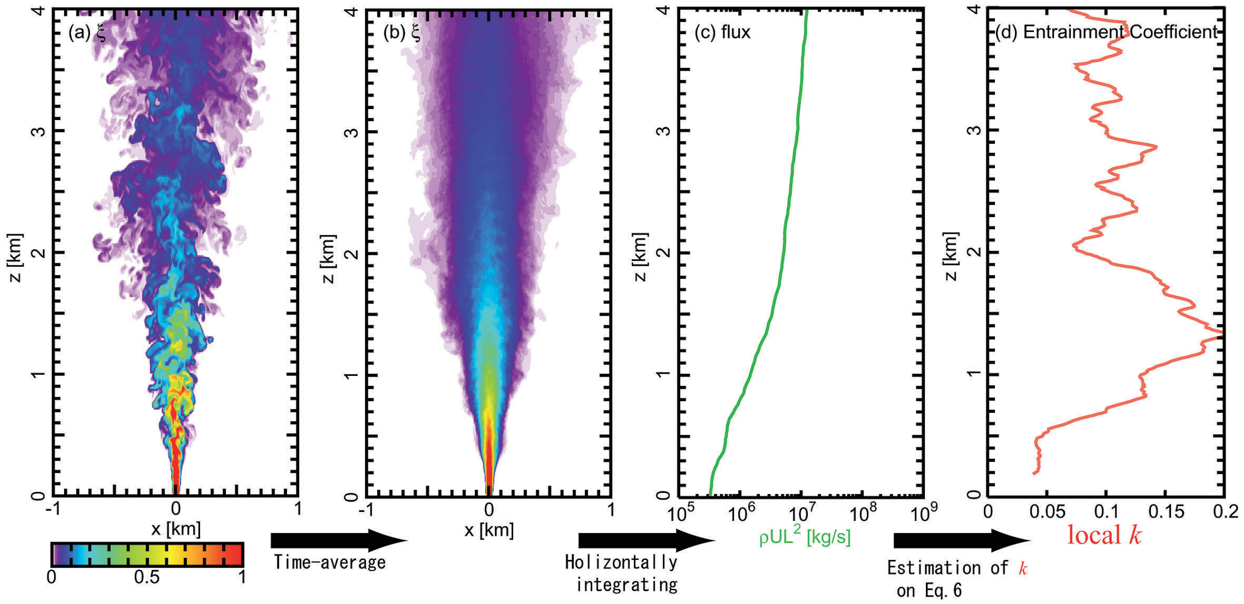

Fig. 2 Procedure of estimation of local *k* on the basis of the unsteady 3-D simulation of eruption cloud for the eruption in the mid-latitude atmosphere with the mass discharge rate of 10^6 kg/s, the initial temperature of 1053 K, and the volatile content of 0.06. (a) Cross-sectional distribution of the mass fraction of magma, ξ, at 214 s from the beginning of the eruption. (b) Cross-sectional distribution of the time-average value of ξ. (c) Vertical profile of the mean mass flux. (d) Vertical profile of the value of local *k*.

teristic quantities as a given downstream distance from the vent (Fig. 2c). Finally, we estimate the value of k at each height, substituting the above values to the definition of entrainment coefficient (Fig. 2d) as

$$\text{local } k = [dQ / dz]/[2\rho_a (M / \rho)^{1/2}], \tag{1}$$

where Q is the mean mass flux, z is the vertical distance from the vent, ρ_a is the atmospheric density, M is the mean momentum flux, and ρ is the mean density, respectively.

Fig. 2d indicates that the value of k is 0.03 – 0.05 below z=1 km, whereas it approaches to 0.1 as the distance increases. These results imply that the efficiency of entrainment for the flow of eruption cloud varies with height, whereas that for a pure jet and plume is almost constant.

4. Intensity of Turbulence in Umbrella Cloud

During explosive eruptions, pyroclasts (volcanic ash and pumice grains) are transported by an umbrella cloud and fall out to the ground surface to form tephra fall deposits in a broad area (Fig. 3) [6]. In previous tephra dispersal models, two extreme relations between turbulence and particle settling in the umbrella cloud have been assumed; pyroclasts are homogeneously mixed inside the umbrella cloud because of turbulence, whereas they fall out at their terminal velocities from the bottom of the umbrella cloud where the turbulence diminishes [e.g., 7, 8, 9]. However, when the intensity of turbulence is not strong enough to homogenize particles inside umbrella clouds, particles are expected to concentrate at the base of the umbrella clouds. In this study we investigate the effects of the intensity of turbulence on the homogeneity of umbrella clouds.

The degree of particle concentration at the base of umbrella cloud (ratio between the particle concentration at the bottom and the mean particle concentration of the entire depth) can be evaluated by a parameter κ, which is expressed as a function of the ratio of terminal fall velocity v_t and the intensity of turbulence W_{rms} as

$$\kappa = [v_t / CW_{\mathrm{rms}}]/[1 - \exp(-v_t / CW_{\mathrm{rms}})], \tag{2}$$

where C (~0.8) is an empirical constant [10]. When W_{rms} is sufficiently large relative to v_t, the value of κ is close to 1. This situation represents homogeneous distribution. On the other hand, when W_{rms} is as small as or smaller than v_t, the degree of particle concentration at the base (i.e., κ) increases with v_t/W_{rms}. We attempt to estimate the value of κ from W_{rms} which is determined by 3-D numerical simulations.

We carried out the 3-D simulation for the magma discharge rate of 10^9 kg/s for vent conditions matching those of the 1991 Pinatubo eruption [6]. From the numerically obtained W_{rms} as well as the theoretically obtained v_t for a given particle diameter d, the value of κ for a given d is calculated using Eq. (2). We can judge whether an umbrella cloud is homogeneous or inhomogeneous on the basis of the values of κ. Fig. 4 shows the distributions of κ for different particle diameters ranging from 1/16 to 16 mm. For d = 1/8 mm or less (Figs. 4a and 4b), κ ~ 1 in the whole eruption cloud. For d = 1 mm, the value of κ is nearly 1 around the central part of the cloud, while it is substantially greater than 1 in the umbrella cloud away from the central axis of the cloud (Fig. 4e). For d = 16 mm, the value of κ is greater than 1 in the whole range of the eruption cloud except for the central part of the cloud (Fig. 4i). These results suggest that particles greater than a few mm in diameter are not homogeneously distributed in the vertical direction, but tend to concentrate at the base of the umbrella cloud. On the other hand, fine ash with d < 1/8 mm is distributed homogeneously in the umbrella cloud.

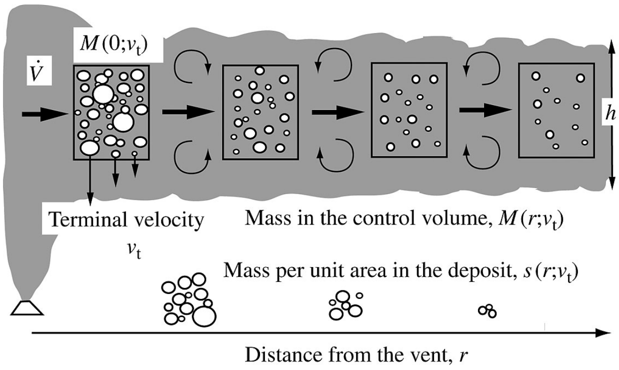

Fig. 3 Schematic illustration of the tephra-fall from an umbrella cloud spreading as a gravity current [6]. Note that this illustration shows a half of the umbrella cloud which radially expands from the central axis above the vent.

5. Summary

We have developed a numerical 3-D model of eruption cloud which can reproduce the fundamental features of eruption column and umbrella cloud and the structures of turbulence. Our numerical simulations explain the variation of the efficiency of turbulent mixing in eruption column and the concentration of particles around the bottom of the umbrella cloud.

6. Acknowledgement

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7. References

[1] A. W. Woods, "The dynamics of explosive volcanic eruptions", Rev. Geophys., 33(4), 495–530, 1995.

[2] Y. J. Suzuki, T. Koyaguchi, M. Ogawa, and I. Hachisu, "A numerical study of turbulent mixing in eruption clouds using a three-dimensional fluid dynamics model", J. Geophys. Res., 110, B08201, 2005.

[3] P. L. Roe, "Approximate Riemann solvers, parameter vectors, and difference schemes", J. Comput. Phys., 43,

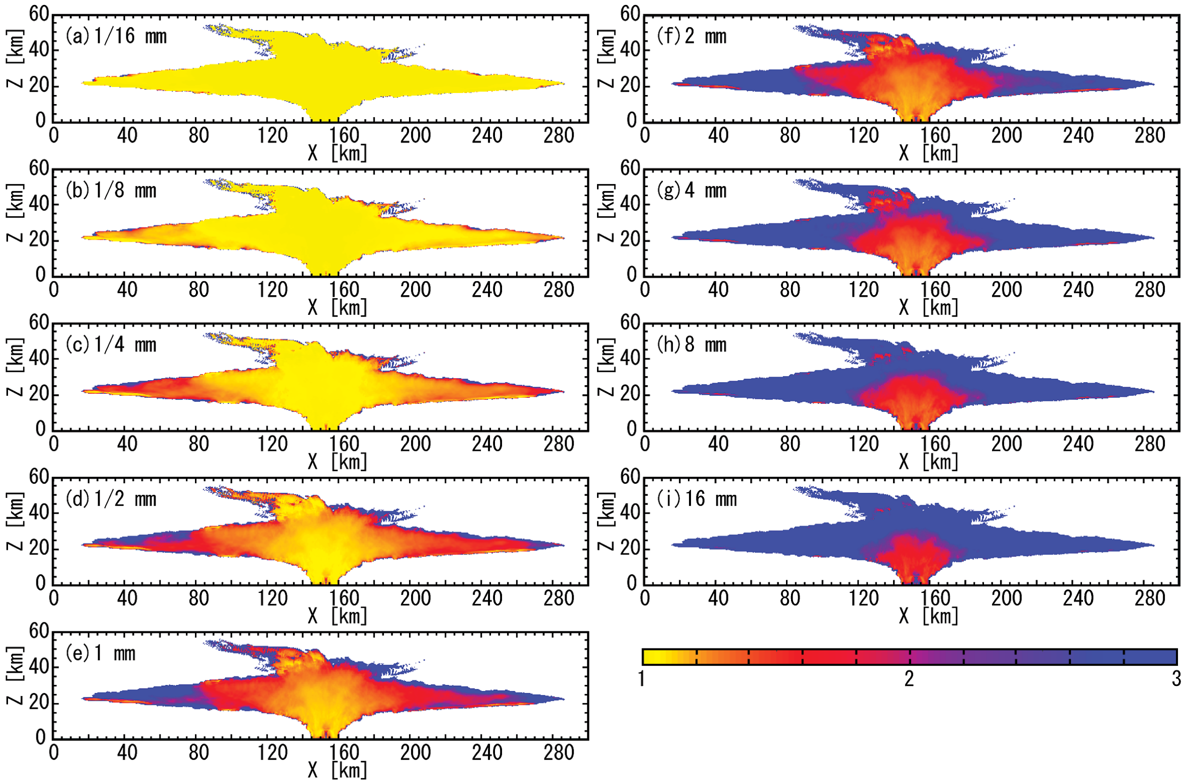

Fig. 4 Distribution of the values of κ for particle diameters of (a) 1/16, (b) 1/8, (c) 1/4, (d) 1/2, (e) 1, (f) 2, (g) 4, (h) 8, and (i) 16 mm in the eruption cloud of the 3-D numerical simulation [10]. The eruption conditions of the climactic phase of the Pinatubo eruption are applied (the mass discharge rate of 10^9 kg/s).

357–372, 1981.

[4] B. van Leer, "Towards the ultimate conservative difference scheme III. Upstream-centered finite-difference schemes for ideal compressible flow", J. Comput. Phys., 23, 263–275, 1977.

[5] B. R. Morton, G. I. Taylor, and J. S. Turner, "Turbulent gravitational convection from maintained and instantaneous sources", Proc. R. Soc. London, Ser. A, 234, 1–23, 1956.

[6] T. Koyaguchi, "Grain-size variation of tephra derived from volcanic umbrella clouds", Bull. Volcanol, 56, 1–9, 1994.

[7] D. Martin, and R. I. Nokes, "Crystal settling in a vigorously convecting magma chamber", Nature, 332, 534–536, 1988.

[8] M. I. Bursik, R. S. J. Sparks, J. S. Gilbert, and S. N. Carey, "Sedimentation of tephra by volcanic plumes: I. Theory and its comparison with a study of the Fogo A plinian deposit, Sao Miguel (Azores)", Bull. Volcanol., 54, 329–344, 1992.

[9] T. Koyaguchi and M. Ohno, "Reconstruction of eruption column dynamics on the basis of grain size of tephra fall deposits. 1. Methods", J. Geophys. Res., 106(B4), 6499–6512, 2001.

[10] T. Koyaguchi, K. Ochiai, and Y. J. Suzuki, "The effect of intensity of turbulence in umbrella cloud on tephra dispersion during explosive volcanic eruptions: Experimental and numerical approaches", J. Volcanol. Geotherm. Res., in subjudice.

爆発的火山噴火における噴煙内部の乱流シミュレーション

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本プロジェクトでは、大規模数値シミュレーションを用いた固体地球と地球表層・大気にまたがる火山現象の理解と計算結果の防災への応用を目指している。特に、火山噴煙の到達高度や火山灰の降灰分布を支配すると考えられる噴煙内の乱流についての基礎的な理解を進めている。

火山噴煙のダイナミクスは、噴出物（火山灰＋火山ガス）と大気からなる混合物の密度変化、乱流による混合過程、大気の成層構造、などの影響によって広いスケール階層性と強い非線形性を持つ。爆発的火山噴火では、火口から火山灰と火山ガスの混合物が高温・高速で噴出する。噴出物は90wt.%以上の火山灰（固体）を含んでいるため、噴出直後は周囲の大気よりも重く、乱流混合で周囲の大気を取り込むと取り込んだ大気を火山灰の熱によって急激に膨張させ、周囲の大気よりも軽くなる。さらに、成層構造を持つ大気は上空に行くほど密度が低くなるため、上空では相対的に噴煙が重くなる。本プロジェクトではこのような非線形な密度変化に起因する噴煙現象を再現するため、混合比によって理想気体の状態方程式における気体定数を変化させた。また、火口付近の流れのスケールと噴煙全体の流れのスケールを同時に再現するため、一般座標系を適用した。その結果、10^6～10^{10} kg/sにわたる噴出率の火山噴煙について定性的・定量的性質を再現することに成功した。

乱流状態の噴流として振舞う噴煙柱では、乱流混合の効率が火砕流の発生条件や噴煙柱の到達高度を決定する。一般に、理想気体の乱流ジェットや乱流プルームでは混合効率の指標となるエントレインメント係数 k は高さ方向にほぼ一定（～0.1）であることが室内実験で示されている。火山噴煙は非線形な密度変化や大気の成層構造を持つため、エントレインメント係数 k が異なる高さで一定値をとるとは限らない。そこで、3次元シミュレーションを基に噴煙柱内のエントレインメント係数 k の高さ方向プロファイルを測定した。その結果、k の値は火口付近で0.03～0.05と小さな値を取り、噴煙柱上部で0.1と理想気体ジェット・プルームとほぼ同じ値をとることが明らかになった。これは、噴煙柱内部で乱流混合効率が変化することを示唆している。

爆発的噴火においては、火山灰粒子が傘型噴煙によって高層大気中で水平方向に運ばれ、その結果として降灰分布が決定される。従来の火山灰拡散・堆積モデルでは、火山灰粒子が傘型噴煙内部で乱流によって均質に混合しつつ底部から終端速度で分離することが仮定されていた。しかしながら、噴煙の乱流強度が弱い場合には、火山灰粒子が傘型噴煙内部で均質化するという仮定が満たされない可能性がある。そこで、3次元シミュレーションを基に乱流強度が粒子の均質化に及ぼす影響を調べた。その結果、ピナツボ火山1991年に相当する噴火（噴出率が10^9 kg/s）では数mmよりも小さい粒子は傘型噴煙内部で均質に分布するが、それより大きい粒子は均質化されず、傘型噴煙下部に濃縮することが示唆された。

キーワード：火山噴煙, 擬似ガスモデル, 乱流混合, 火山災害