Simulation Study on the Dynamics of the Mantle and Core in Earth-like Conditions

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Thermal convection in the mantle and the outer core is the origins of various Earth's activities and essentially important in the Earth's evolution. We investigated styles of convection in the mantle by using a three-dimensional spherical-shell code that includes effects of phase transitions, temperature-dependent viscosity with plastic yielding near the surface, and viscosity variations in the mantle. We find that, these effects spontaneously reproduce plate-like behaviors and slab stagnation around the transition zone, with appropriate value of viscosity increase in the lower mantle. In the core convection, one of the most important properties is the extremely low viscosity of the fluid. Our high-resolution model for geodynamo successfully simulated torsional oscillations, and we analyzed their details by comparing with theories and observed short-term geomagnetic field variations. On the other hand, we apply a high-order compact difference scheme to treat such thin boundary layers with smaller computational recourses. As a result, faster convergence is achieved and we confirmed the validity of the scheme. In addition, study of turbulence in liquid metal convection is within our scope. Our numerical code on thermal convection with the material properties of a liquid metal can reproduce the character of motion observed by laboratory experiments.

Keywords: mantle convection, slab stagnation, core convection, geodynamo, geomagnetic secular variation, convection of low Prandtl number fluid

1. Introduction

Our group is composed of two subgroups, aiming for comprehensive understanding of the dynamics of the Earth's mantle and core as a combined solid-Earth system. The mantle convection group focuses on dynamical behaviors of the Earth's mantle and simulates infinite-Prandtl-number thermal convection. Particular attention has been paid on integrating realistic mantle properties (e.g., variable viscosity, phase transition, plate behaviors) into the model and reproducing the images obtained from seismic tomography. The geodynamo group simulates thermal convection of the fluid outer core and a resultant generation process of the geomagnetic field. In order to reach the core conditions, we have made attempts to reduce viscous effects in the dynamo model by decreasing the Ekman number ($E = \nu/2\Omega r_0^2$; ν: kinematic viscosity, Ω: Earth's angular velocity, r_0: core radius), the Prandtl number ($Pr = \nu/\kappa$; κ: thermal diffusivity), and the magnetic Prandtl number ($Pm = \nu/\eta$; η: magnetic diffusivity). We are also studying the nature of turbulence in liquid metal convection by comparing the results of numerical simulations with laboratory experiments.

2. Simulations of mantle convection

The Earth's mantle is composed of solid rocks but it flows like a viscous fluid in a geologic time scale. This convective flow of the mantle is emerging as the motion of tectonic plates on the Earth's surface. The motion of surface plates causes earthquake, volcanism and mountain building at the plate margins. As the mantle flow transports the heat from the hot interior, the whole of the Earth has been cooling through its history. It also controls the boundary conditions of the outer core. Hence, mantle convection is the key process for understanding the activity and evolution of our planet. Seismic tomography reveals the natural mode of convection in the Earth is whole mantle with subducted plates (slabs) clearly seen as continuous features into the lower mantle. The Earth's mantle is characterized by the coexisting state of slabs stagnating around the transition zone and falling into the lower mantle [1].

We simulated fully dynamical and self-consistent thermal convection in high-resolution 3-D spherical shell models which range up to Earth-like conditions in Rayleigh number, and succeeded in spontaneous generation of plate-like behavior with slab stagnation [2]. We examined the influence of three factors: phase transitions, temperature dependent viscosity with plastic yielding at shallow depth, and viscosity increase in the lower mantle, and clarified the condition for generating stagnant slabs. The temperature dependent viscosity with plastic yielding spontaneously produces plate-like behavior with very localized convergence zones at the surface. This plate-like structure can stagnate in the transition zone with the combination of 660 km phase transition and viscosity increase in the lower mantle. The model including these three factors with adequate values generates the coexisting state of stagnant and penetrating slabs around the transition zone, which are characteristics of mantle convection revealed by seismic tomography (Fig. 1). The key mechanism to generate stagnant slabs is the partly decoupled state of the upper and lower mantle flow due to the phase transition. Behaviors of subducted plates are sensitive to the viscosity increase in the lower mantle. We examined several cases by varying the reference viscosity structure through the mantle (Fig. 2). We confirmed that it is necessary more than 40 times of viscosity increase in the lower mantle to form large-scale stagnation. With smaller value of increase, small-scale stagnations are realized, which is different from the views of mantle tomography. If the value of the viscosity increase exceeds 100 times, the behavior of the slab is weakly depends on the value. The steepness of the viscosity increase is also important for the behavior of subducted slabs. If the viscosity increase is more gradual, the range of stagnation depth is

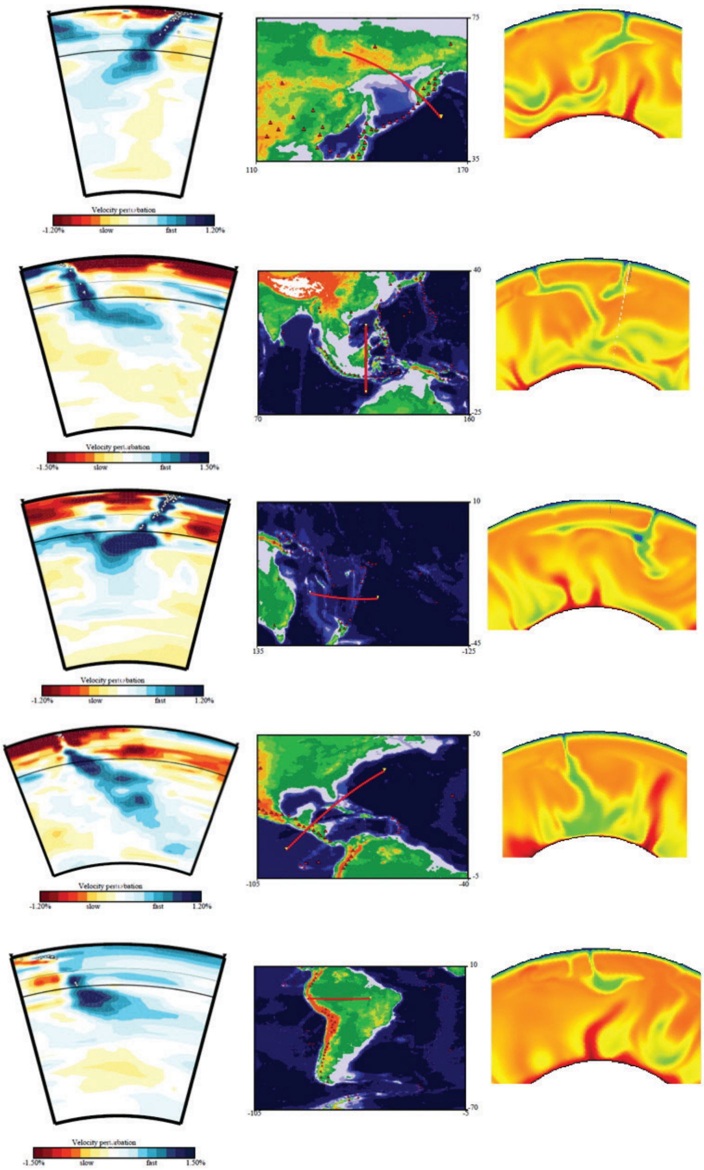


Fig. 1 Various styles of subducted slabs in the mantle. (left) Images from mantle tomography by [3] (blue: high velocity, red: low velocity), (center) the location of each cross-section. (right) Similar styles of subduction reproduced in our 3-D spherical shell convection model containing phase transitions, viscosity layering, and plastic yielding near the surface (blue: low temperature, red: high temperature). The Earth's mantle is characterized by the coexisting states of stagnant and falling slabs, and our numerical model spontaneously reproduces these features.

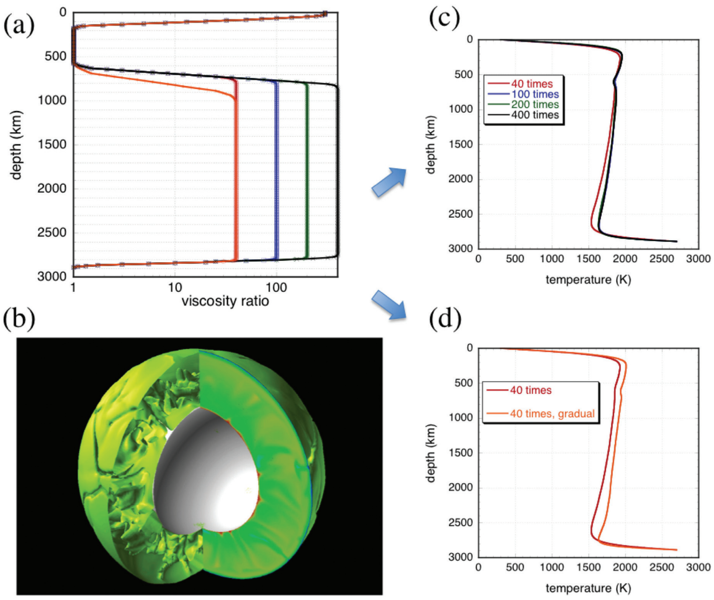


Fig. 2 Comparison among the setting of viscosity profiles in the mantle. (a) Settings of reference viscosity at each depth. The upper mantle viscosity is kept constant, and the lower mantle viscosity is increased from 40 to 400 times. (b) An example of 3-D structure shown by temperature for the case with 40 times viscosity increase. (c) Horizontally averaged temperature profiles. The degree of stagnation is reflected by the difference of the lower mantle temperature and the existence of inflection points around 660 km depth. The effect of viscosity increase is not so large when the viscosity increase exceeds 100 times. (d) Same as (c) but for the comparison of the steepness of the viscosity increase with 40 times. If the viscosity increase is more gradual, the depth range of stagnation becomes broader and the average temperature of the lower mantle increases.

broader. We can elucidate the viscosity structure of mantle that is not clearly understood yet, by quantitatively comparing the result with these viscosity profiles and the images of seismic tomography.

3. Geodynamo simulations

3.1. Torsional waves

We have made attempts to decrease viscosity of the model fluid for the Earth's outer core in order to better simulate core thermal convection and dynamo process [4,5,6]. Our lowest-viscosity model is now capable of decreasing the Ekman number to O(10^{-7}) and the magnetic Prandtl number to O(0.1). Reduction of viscosity brings dramatic changes to flow and magnetic field structures. For example, the boundary condition for the core surface temperature becomes a more important factor for the system to be an Earth-like strong-field dynamo [7]. Low-viscosity geodynamo models make it possible to study geomagnetic field variations of short timescales. Theoretical studies indicate that a field variation obeys a wave-like equation when the system is close to a Taylor state, where both viscous diffusion and inertia have negligibly small effects and the azimuthal component of Lorentz force is zero when averaged over $C(s)$, the side surface of a cylinder of radius, s, coaxial with the rotation axis. The resultant torsional oscillations, which travel in the cylindrically radial (s-) direction with an Alfven-wave speed proportional to the s-component of the magnetic field, have been recognized to be one of the most important origins of decadal field variations [8]. Our recent model well justifies this theory. Using a uniform-heat-flux condition for the surface temperature, we succeeded in producing a strong-field dynamo, in which the magnetic field is largely generated by a large-scale flow in contrast to other uniform-surface-temperature models that fail to drive large-scale flows and sustain strong magnetic fields [5,9]. Viscous diffusion does not affect the primary force balance and the fluid domain outside the inner-core tangent cylinder is in a Taylor state to a good approximation; within 1% in our definition (see Fig. 3 (a)). The degree of Taylorization is better than previous low-viscosity geodynamo models [4]. The time-averaged zonal flow outside the tangent cylinder is westward and particularly stronger near the equator. For a wave analysis, the fluctuating part of the zonal flow, which is almost independent of the height, z, from the equatorial plane, is integrated over $C(s)$ to obtain $V(s,t)$ as a function of radius and time. This fluctuating zonal flow is further transformed to two-dimensional Fourier modes and decomposed into two components that travel toward the equator and toward the rotation axis (the stationary component is negligibly small because of conservation of angular momentum). Figure 3 (b) shows results of our wave analysis. At a fixed radius outside the tangent cylinder, the zonal flow turns eastward and westward in an oscillatory fashion. The phase travels both inward and outward. We calculated the averaged magnetic stress in a similar way and found that the Lorentz force acted as a restoring force for the oscillatory fluctuations of the zonal flow. These results suggest that our solution is basically in a Taylor state and the fluctuating fields behave as torsional waves, at least outside the

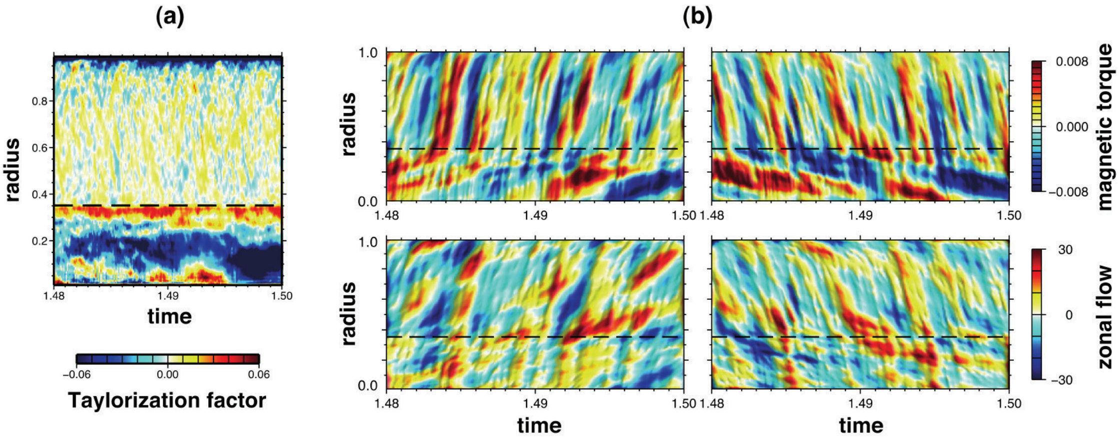


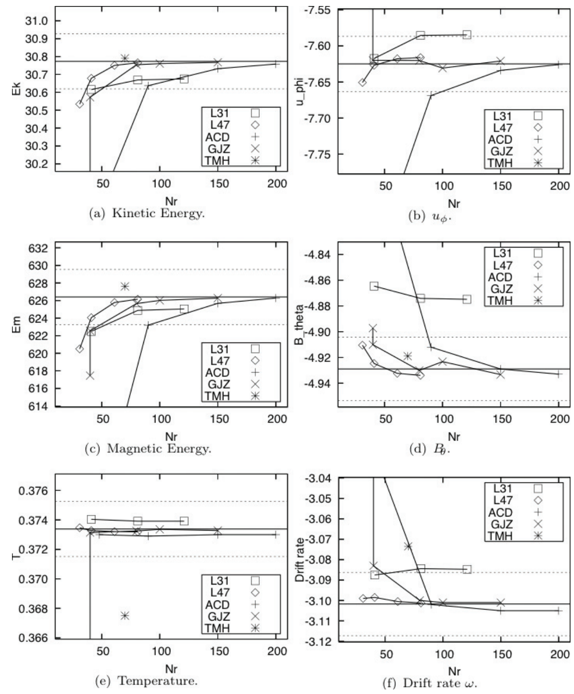
Fig. 3 (a) A Taylorization factor defined as the ratio of the integral of F_ϕ, the azimuthal component of Lorentz force, over $C(s)$, the side surface of an axial cylinder of radius, s, to the integral of the absolute value of F_ϕ over the same surface. Time is scaled by a magnetic diffusion time. Broken line represents the inner core radius. (b) Shown are the time variations of the magnetic torque, BsB_ϕ, and the azimuthal component of velocity, V_ϕ, integrated over $C(s)$. Both integrated BsB_ϕ and V_ϕ represent fluctuations from their time averages and have been decomposed into outgoing (left) and ingoing (right) components by Fourier analysis. The actual fluctuation is the sum of these two components.

tangent cylinder, as predicted by previous theory.

Gillet and coauthors recently reported that the observed geomagnetic data suggested faster propagation of torsional waves than previously estimated and its cause could be attributed to a stronger interior magnetic field [10]. They also showed that the propagation direction was primarily outward, which is totally different to our calculation. Although a direct comparison to the Earth's core convection is too premature, our solution seems to be still crude to represent ideal torsional waves. For example, there is a trend of slower outgoing propagation in the zonal flow at around t = 1.491 and s = 0.7. This signature cannot be explained by Alfven waves and is probably caused by a local effect of advection. In our model, the magnetic energy density is only ten times greater than the kinetic energy density on average, whereas this ratio, considered to be an index of magnetically dominated strong-field dynamo, is expected to be at least several hundreds in the Earth's core. Simulations of higher-resolution and lower-viscosity geodynamo models are still needed to reach the core condition and to make a comparison to the geomagnetic data.

3.2. Implementation of a high-order scheme

As noted in the previous section, the currently most advanced models are run at E = O(10^{-7}). At such a low-Ekman number, a very sharp Ekman boundary layer with radial thickness of O($E^{1/2}$) develops near the inner and outer boundaries of the core. Numerical models must be capable of resolving such thin boundary layers. Since a spectral approach using Chebyshev expansion is, in general, not very good at representing a very sharp structure, we have investigated finite difference discretization for low-Ekman-number simulations. However, in ordinary finite difference discretization, fine mesh and many stencils are needed to yield solutions of acceptable accuracy. As a result, a lot of memory space and computing time may be consumed. One approach to perform numerical dynamo simulations at a low-Ekman number with high accuracy and less computational cost is to use higher order discretization scheme, which use coarser mesh to yield solutions of comparable



(a) Kinetic Energy. (b) u_ϕ. (c) Magnetic Energy. (d) B_θ. (e) Temperature. (f) Drift rate ω.

Fig. 4 Convergence behavior of the solutions with respect to radial resolution N_r: (a) kinetic energy, (b) azimuthal velocity, (c) magnetic energy, (d) axial magnetic field, (e) temperature, and (f) drift rate. The present results are classified by L31 and L47 corresponding to spherical harmonic expansion up to 31 and 47, respectively, while ACD, GJZ and TMH denote results from different codes. Horizontal solid lines show the standard values and horizontal dashed lines represent the deviation by 0.5% from the standard values.

accuracy relative to the lower order discretization scheme using finer mesh. A combined compact difference scheme (CCDS) can achieve a high-order accuracy and good spectral resolution with a small stencil. We apply a high-order three-point CCDS in the radial direction to problems of thermal convection and convection-driven dynamo in a rotating spherical shell [11]. To evaluate accuracy of the CCDS, we have solved the benchmark problems. It is confirmed that accuracy better than 1% is achieved with the CCDS even with a modest number of grid mesh. Quantitative comparison with other finite difference schemes indicates that the CCDS is superior to others using more stencils. As a result, faster convergence behavior of the CCDS is observed in most quantities with an accuracy of 0.5% (Fig. 4).

3.3. Coherent structure with oscillation in liquid metal convections

The study on the nature of thermal convection in low Prandtl number fluids is essential for the dynamics of the Earth's outer core, and the difference of the flow behavior from Pr~1 fluids like water and air is very important. In lower Pr fluids, the two-dimensional steady roll structure emerging at the onset of convective flow easily becomes time-dependent just above the critical Rayleigh number (Ra), and theoretical studies propose oscillatory instability such as "traveling-wave convection" in the direction of the roll axis [12]. Transition to turbulence with increases in Ra in low Pr fluids occurs at much lower Ra than water or air, and large-scale flow is also expected to emerge easily.

Our laboratory experiments on thermal convection with liquid metal by using an ultrasonic velocity profile measurements visualized the flow pattern in a gallium layer with simultaneous measurements of the temperature fluctuations, from 10 to 200 times above the critical Ra [13,14]. It was made in a non-rotating rectangular container. In those experiments, the presence of a roll-like structure with oscillatory behavior was established (Fig. 5), even in the Ra range where the power spectrum of the temperature fluctuation shows features of developed turbulence. The flow structure was interpreted as a continuously developed one from the oscillatory instability of two-dimensional roll convection around the critical Ra. It was shown that both the velocity of the flows and the frequency of

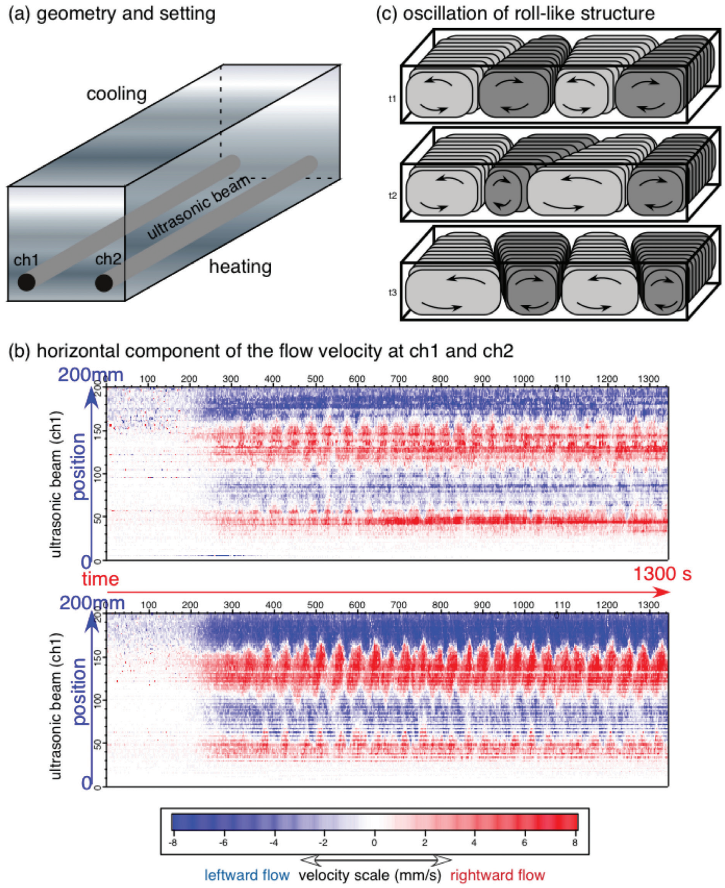


Fig. 5 Setting and result of ultrasonic flow velocity measurements for Rayleigh-Bénard convection in liquid gallium (laboratory experiment). (a) Geometry of the container and setting of the measurement beam lines in liquid gallium. (b) Examples of the velocity profiles for the case Ra = $8x10^4$; horizontal axis is the elapsed time, vertical axis is the position, and color maps indicate the horizontal component of the convective flow velocity. The vertical temperature difference was set at the time = 200 s, and convection pattern emerged after that. Four clusters of velocity with periodic oscillations are clearly observed. (c) Interpretation of the global flow pattern. Oscillatory roll-like structure exists in the vessel, and the oscillation period is comparable to the circulation time of the flow in a roll.

the oscillation increase proportional to the square root of Ra, and that the oscillation time of the roll structure is comparable to the time to complete one circulation of the flow.

We made up a code for numerical simulation of thermal convection to compare with the results obtained by the laboratory experiments. Furthermore, we analyzed the fine scale structure and short time variation relating to turbulence, those are difficult to obtain by laboratory experiments due to the limitation of measurements. The numerical simulation is performed for three dimensional rectangular box, with no-slip boundary conditions at all boundaries, fixed temperature at the top and bottom, and insulating at side walls. The range of Ra for numerical simulations is from critical value to 200 times above it. The material properties of the working fluid are those of liquid gallium and Pr=0.025. We used enough grid points to resolve the small-scale behavior without any assumption for the turbulence. Our numerical result reproduced oscillatory convection patterns as observed in the experiments. Statistical values, such as the relation of the circulation time and oscillation period, Rayleigh number dependence of the mean velocity and the oscillation frequency, are good agreement in both laboratory and numerical studies (Fig. 6). This confirms that both of our laboratory experiment and numerical simulation are reliable ones. The series of numerical simulations with the increase in Ra revealed the onset point of oscillatory convection and subsequent transition to turbulence. The power spectrum densities calculated from the velocity and temperature dataset clearly indicate the feature of low Pr fluid, that is, temperature is more diffusive than momentum and the corner frequency is higher for velocity spectrum in the region of developed turbulence.

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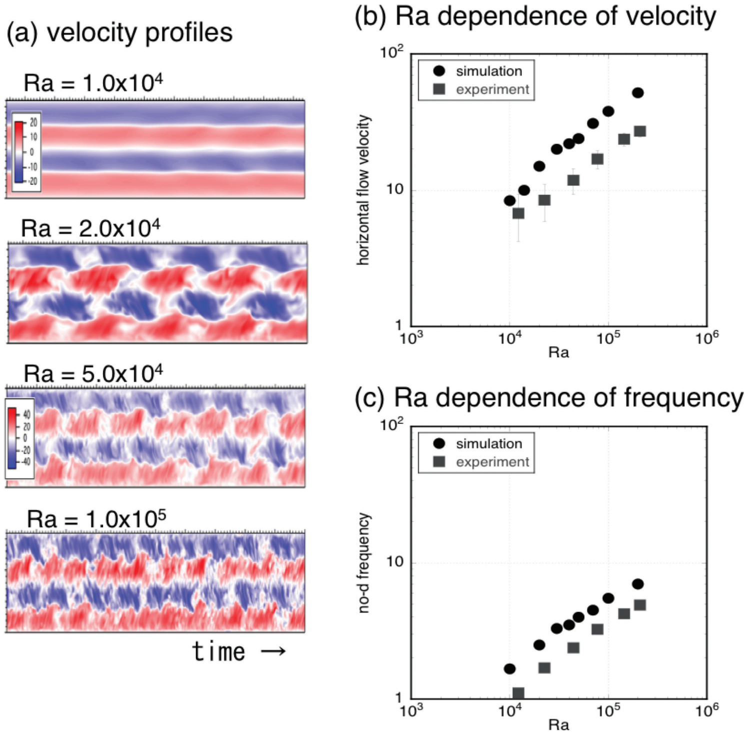

Fig. 6 Quantitative comparison between laboratory experiments and numerical simulations. (a) Velocity profiles by numerical simulations showing Rayleigh number dependence. Oscillatory four-roll structures similar to the laboratory experiments are reproduced. (b) Comparison of non-dimensional flow velocity. (c) Comparison of non-dimensional frequency of the pattern oscillation. These are good agreement between simulations and experiments.

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実地球環境でのマントル・コア活動の数値シミュレーション

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地球のマントルとコアで起こっていると考えられる熱対流は、固体地球に生起するさまざまな活動の根本原因であり、地球の進化を知る上で重要である。本プロジェクトではマントルとコアの対流を可能な限り地球に近い条件で取り扱い、実際の現象との比較をおこなってきた。

マントルの対流については、すでに開発済みの有限要素法を用いた球殻熱対流コードで実行した。粘性の温度依存性と表面付近での降伏応力の導入によって、自然にプレート的な運動が再現される条件で、上部マントルと下部マントルの粘性比を等倍から400倍まで変えて沈み込むプレートの挙動を調べた。結果、粘性比が40倍を超えると、マントル遷移層でプレートの横たわり（滞留）が顕著に見られるようになった。さらに粘性比を大きくしても横たわりの形態にあまり変化はなく、粘性比そのものよりも粘性増加の勾配が形態に大きく影響することが分かった。代表的なケースについて30億年以上の時間積分をおこない、プレート配置が長時間にわたって変動し続ける結果を得ることに成功した。本結果を用いて、地震波トモグラフィーで得られた沈み込んだプレートの形状との比較を多くの沈み込み帯についておこなった。

コアの対流については、地球ダイナモのシミュレーションを球関数展開とチェビシェフ多項式展開とに基づくスペクトル変換法を用いた既に開発済みのコードで実行した。結果、低い粘性のもとで理論的に予想されているコアのねじれ振動が、シミュレーションにおいても、内向き・外向きの進行波として存在していることを確認した。さらに、既存モデルで採用していたブシネスク近似から、ゼロでない断熱温度勾配の効果を考慮する近似に変更し、コードの検証をおこなった。同様の離散化を用いたコードで、コア表面に安定成層が存在する条件でのダイナモシミュレーションを比較的粘性の低い領域でおこなった。一方であらたに、動径方向に対する離散化を高次の結合コンパクト差分法で扱うコードを開発し、チューニングをおこなうとともにダイナモベンチマーク問題を解き、手法の有効性を確認した。我々は同時に、コア乱流のモデル化に関する研究を進めており、熱対流の数値シミュレーションでレイリー数を変えたパラメータスタディを液体金属の実際のプラントル数 O (10^{-2}) を用いて実行し、室内実験で得られている大規模流の変動と乱流の特徴を再現することに成功した。

キーワード: マントル対流, スタグナントスラブ, コア対流, 地球ダイナモ, 地磁気永年変化, 低プラントル数の対流