The 4th Research Meeting of Ultrahigh Precision Meso-Scale Weather Prediction
Kobe, Japan
2014/03/07

# Mesoscale Weather Prediction with a hybrid EnKF-4DVAR system

Kosuke Ito

JAMSTEC/MRI

■ Dr. Masaru Kunii kindly provided the results of EnKF

# Concept of 4DVAR

- Cost function

Misfit b/w init. value of analysis and first guess

$$J = \boxed{\frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_\mathrm{b})^\mathrm{T}\mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_\mathrm{b})}$$
$$+\boxed{\frac{1}{2}\sum_t(\mathbf{y}_t - \mathbf{H}\mathbf{x}_t)^\mathrm{T}\mathbf{R}^{-1}(\mathbf{y}_t - \mathbf{H}\mathbf{x}_t)}$$

Misfit b/w analysis and observation

- Optimization problem

  Find $\mathbf{x}_0$ that gives the minimum value of $J$
  (with the constraint of the model dynamics
  $\delta\mathbf{x}_t=\mathbf{M}\delta\mathbf{x}_0$ within data assimilation window)

# Solution of BLUE (Johnson et al. 2005)

- The solution of 4DVAR and KF is the same at the end of DA (Linearity, Gaussianity, same **B** and **R** assumed)

$$\mathbf{x}_{a,t} = \mathbf{x}_{b,t} + \mathbf{MBM}^T\mathbf{H}^T\left(\mathbf{R} + \mathbf{HMBM}^T\mathbf{H}^T\right)^{-1}(\mathbf{y} - \mathbf{H}\mathbf{x}_{b,t})$$

- Consider the singular value decomposition below

$$\mathbf{L} \equiv \mathbf{R}^{-1/2}\mathbf{HMB}^{1/2} = \mathbf{UDV}^T = \sum_{l=1}^{m} \mathbf{u}_l \lambda_l \mathbf{v}_l^T$$

observability matrix

- Then, the solution can be rewritten as

$$\mathbf{x}_{a,t} = \mathbf{x}_{b,t} + \mathbf{MB}^{1/2}\sum_{l=1}^{m}\frac{\lambda_l^2}{1+\lambda_l^2}\frac{\mathbf{u}_l^T\mathbf{R}^{-1/2}(\mathbf{y} - \mathbf{H}\mathbf{x}_{b,t})}{\lambda_l}\mathbf{v}_l$$

# SVD decomposition of analysis increment

(Linearity and Gaussianity assumed; Johnson et al. 2005)

$$\mathbf{L} \equiv \mathbf{R}^{-1/2}\mathbf{HMB}^{1/2} = \mathbf{UDV}^T = \sum_{l=1}^{m} \mathbf{u}_l \lambda_l \mathbf{v}_l^T$$

observability matrix

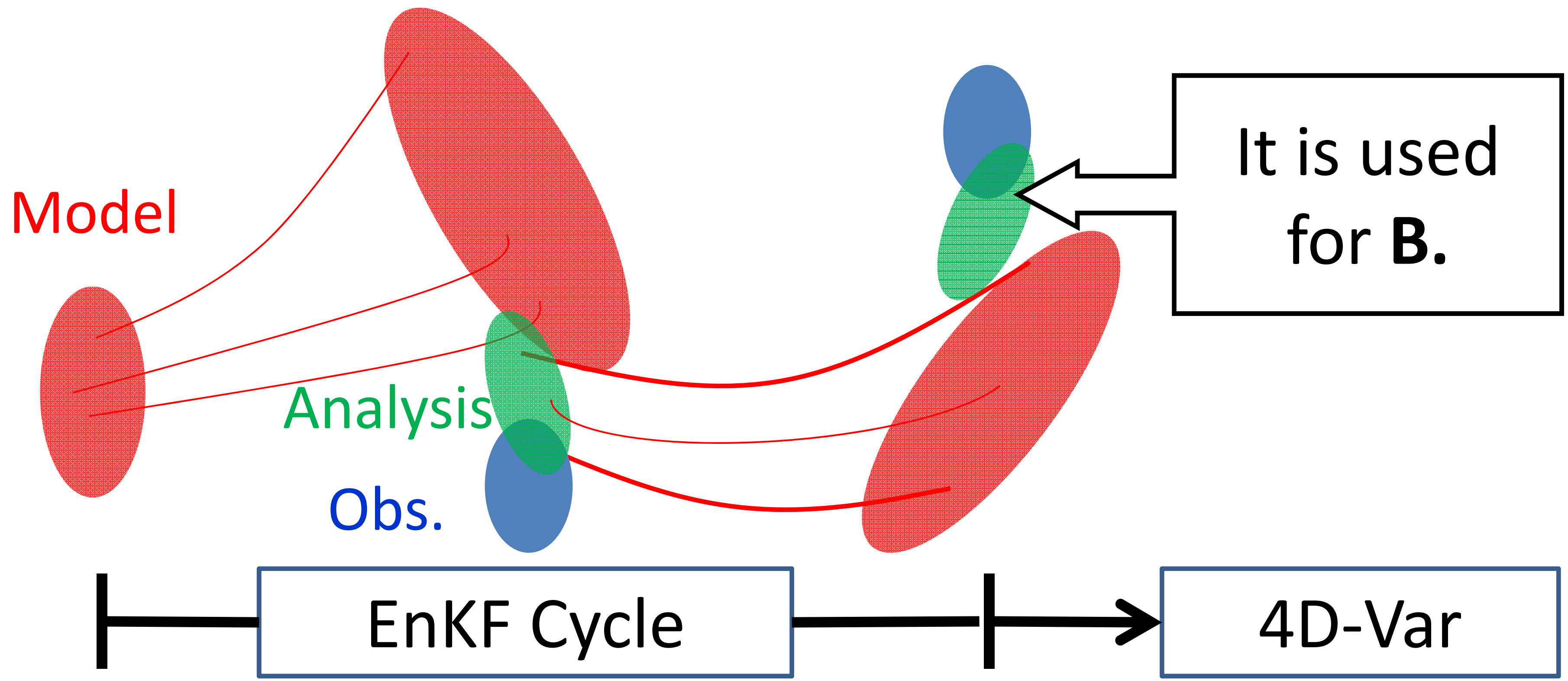
$$\mathbf{x}_{a,t} = \mathbf{x}_{b,t} + \underbrace{\mathbf{MB}^{1/2} \sum_{l=1}^{m} \boxed{\frac{\lambda_l^2}{1+\lambda_l^2}} \frac{\mathbf{u}_l^T \mathbf{R}^{-1/2}(\mathbf{y} - \mathbf{H}\mathbf{x}_{b,t})}{\lambda_l} \boxed{\mathbf{v}_l}}_{\text{analysis increment}}$$

- <u>$\mathbf{v}$ is the eigenvector of $\mathbf{LL}^\mathrm{T}$</u>
  →If $\mathbf{H}=\mathbf{R}=\mathbf{I}$, $\mathbf{v}$ is the eigenvector of $\mathbf{MBM}^\mathrm{T}$.
- $\lambda^2 \gg 1$ → singular vector constitutes the solution
  $\lambda^2 \ll 1$ → singular vector does not constitute the solution
- If $\mathbf{R}_{ij}$ is small, $\lambda$ is large.

# Hybrid EnKF-4DVAR (Lorenc 2003)

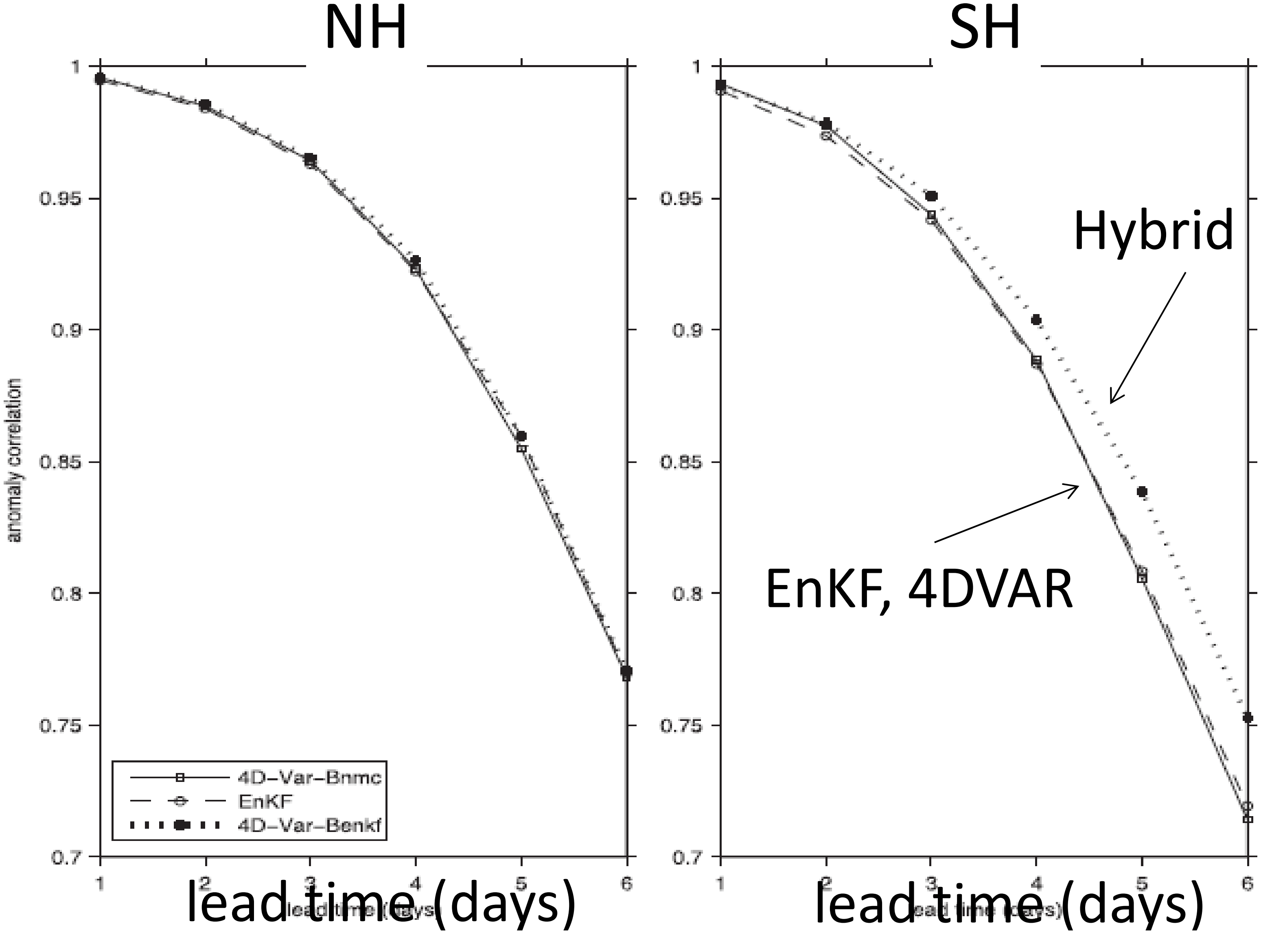
$$J = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b) + \frac{1}{2}\sum_{t'}(\mathbf{x}_{t'} - \mathbf{y}_{t'})^T \mathbf{R}^{-1}(\mathbf{x}_{t'} - \mathbf{y}_{t'})$$

- Conventionally, NMC-based **B** (climatological) is used, but EnKF-based **B** is preferable to represent flow-dependency.



# Hybrid EnKF-4DVAR system

## Anomaly correlation (Buehner et al., 2010)



# Why EnKF-based **B** is needed for severe weather prediction using 4DVAR?

- NMC-based **B** represents only the climatological correlation and is not suitable for severe weather.
- **B** has large influence on the structure of analysis increment away from observational data.
  --> EnKF-based **B** helps to cover sparseness of observations to capture the phenomena of interest.

# Implemementation (Lorenc 2003, Wang et al. 2007)

For simplification, I don't consider the mixture of NMC-**B** here.

- Since $\mathbf{B}^{-1}$ is not easy to get, we use transformed variables **v** defined $\delta\,\mathbf{x}=\mathbf{B}^{1/2}\mathbf{v}$. It simplifies $J_B=(1/2)\mathbf{v}^T\mathbf{v}$ and $\partial J_B/\partial\mathbf{v}=\mathbf{v}$.
- We can obtain **v** with $\mathbf{v}=0|_{iter=1}$, $dJ/d\mathbf{v}$.

| | w/o localization | w/ localozation (α-vector method) |
|---|---|---|
| **B** | $\mathbf{X}\mathbf{X}^T$ [n x n] | $\mathbf{X}\mathbf{X}^T \circ \mathbf{S}$ [n x n] |
| $\mathbf{B}^{1/2}$ | $\mathbf{X}$ [n x m] | $(\mathrm{diag}(\mathbf{x}_1)\mathbf{S}^{1/2},\ldots,\mathrm{diag}(\mathbf{x}_m)\mathbf{S}^{1/2})$ [n x mn] |
| $dJ/d\mathbf{v}$ | $\mathbf{B}^{1/2}dJ/d\delta\mathbf{x}$ [m] | $\mathbf{B}^{1/2}dJ/d\delta\mathbf{x}$ [mn] |
| $\delta\mathbf{x}$ | $\mathbf{B}^{1/2}\mathbf{v}$ [n] | $\mathbf{B}^{1/2}\mathbf{v}$ [n] |

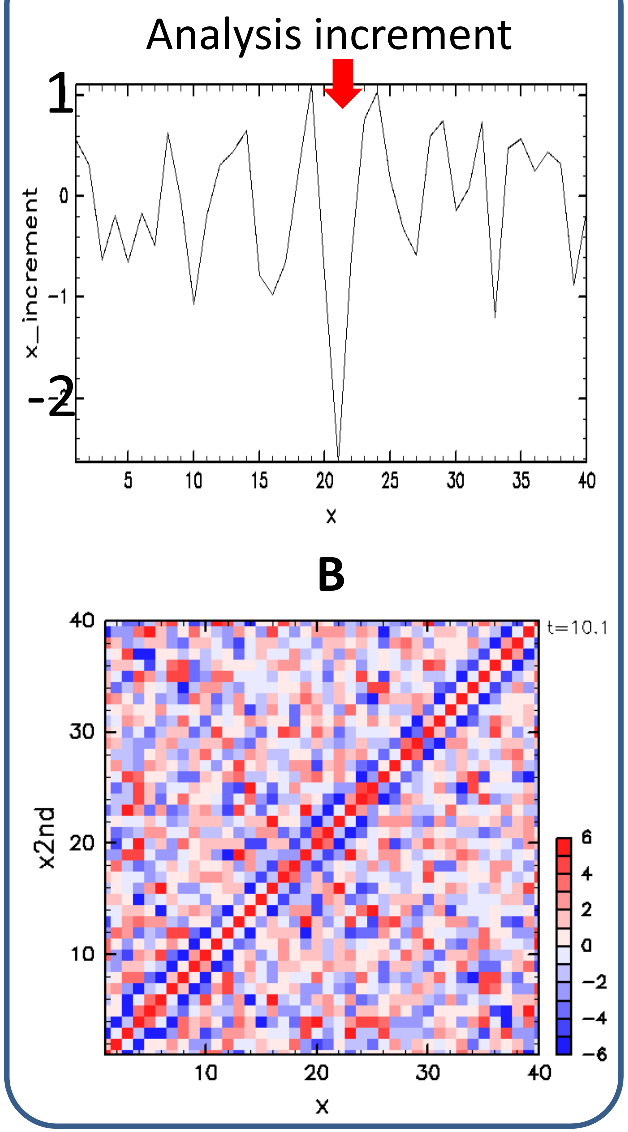
[ ]:size of vector or matrix, $\circ$: Schur product

**X**: perturbations relative to ensemble mean for all members

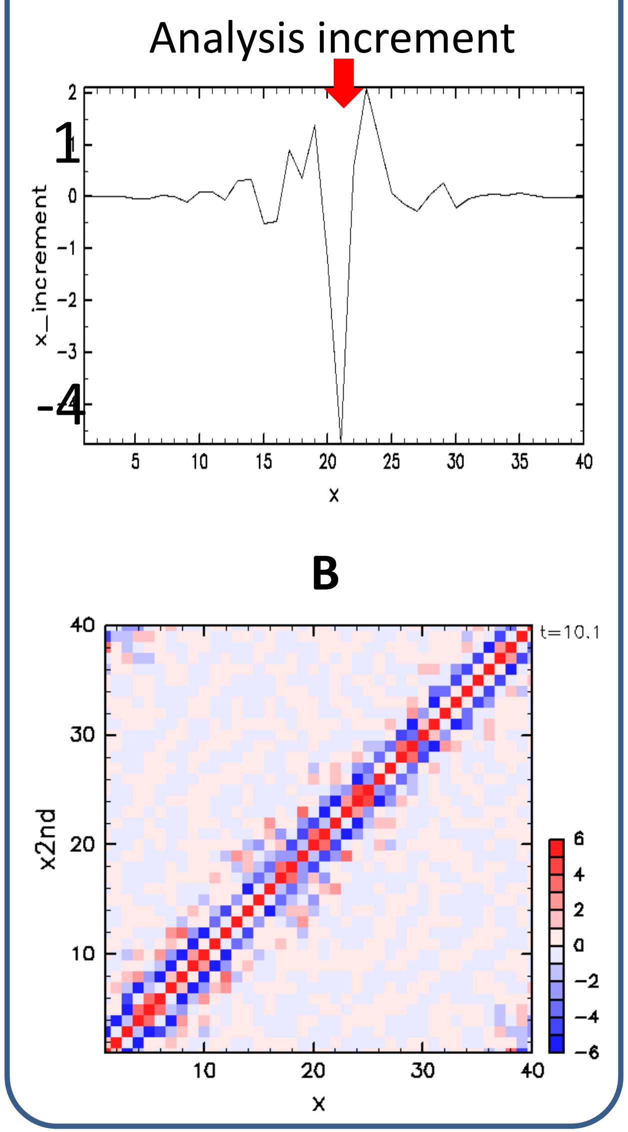
n:degree of freedom, m: num of members, **S**: Localization operator

# Localization (Lorenz96): 1-obs. test

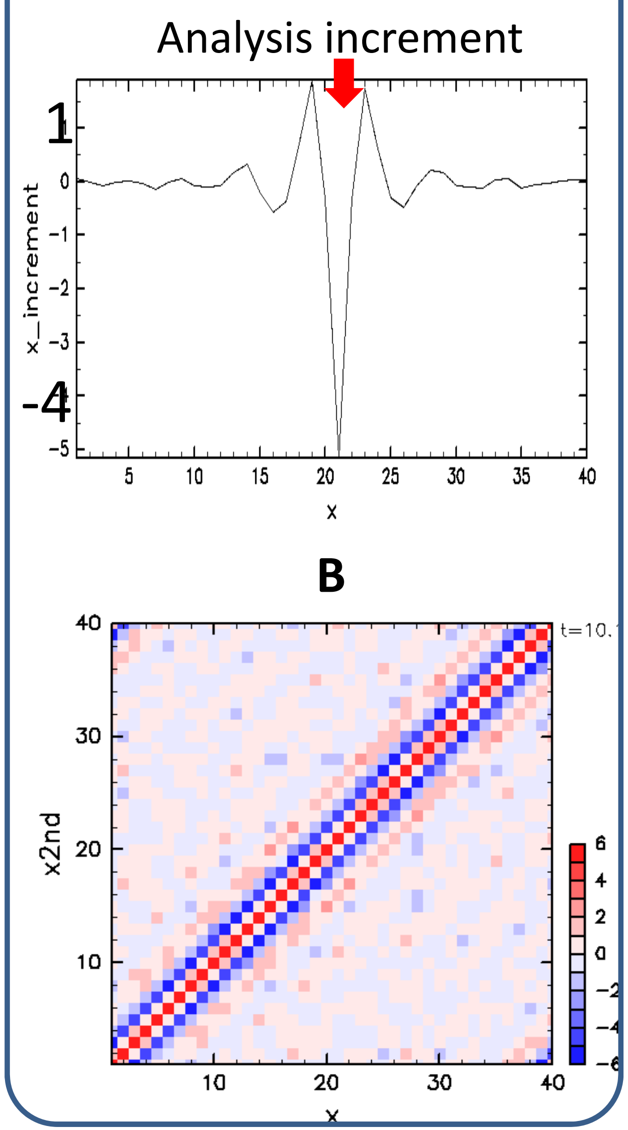
30 mem. No localization




30 mem. w/localization




1000 mem. No localization



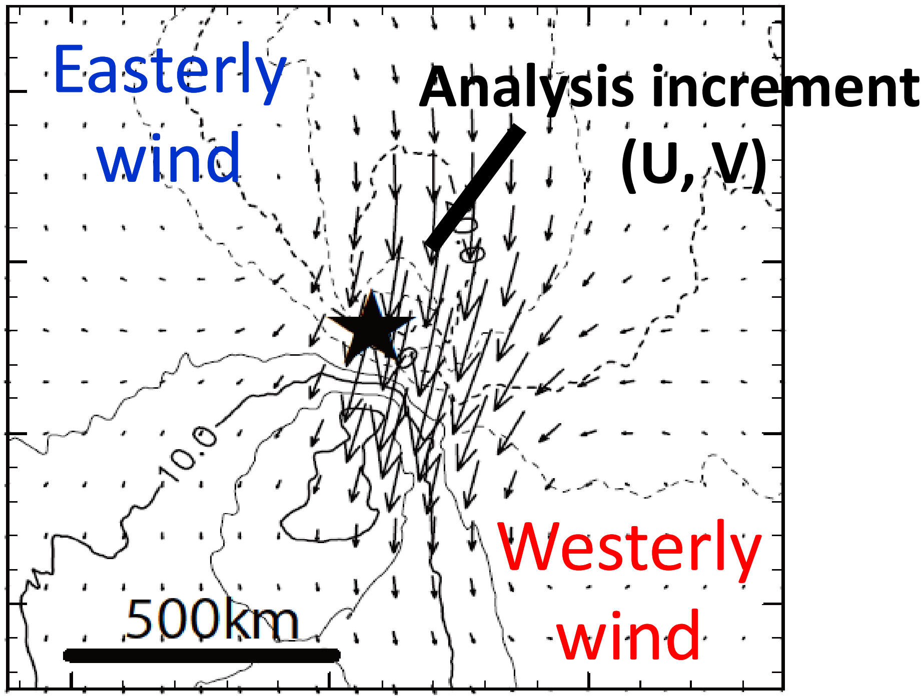
# Implementation to JNoVA

- JNoVA is “JMA Nonhydrostatic model”-based Variational data assimilation system used for the regional forecast in JMA.
- Adjoint-based 4DVAR
- Flow-dependent **B**: EnKF-based 51 members
- Inner model: ⊿x=15km, Outer model: ⊿x=5km
- Large scale condensation
- Assimilation window: 3 hours.
- For simplicity, we show the increment of horizontal wind only.

# 1-obs DA around Typhoon Talas

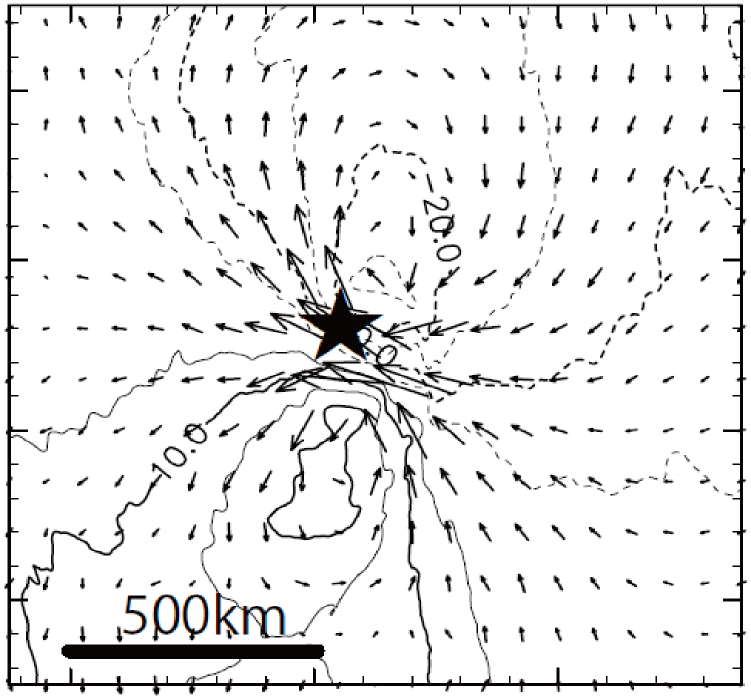
- Obs: U (t=1h, z=1120m) indicated by ★
- Contour : background zonal wind
- Vector: analysis increment of hor. wind (z=1120m)

NMC-based **B** (Conventional)



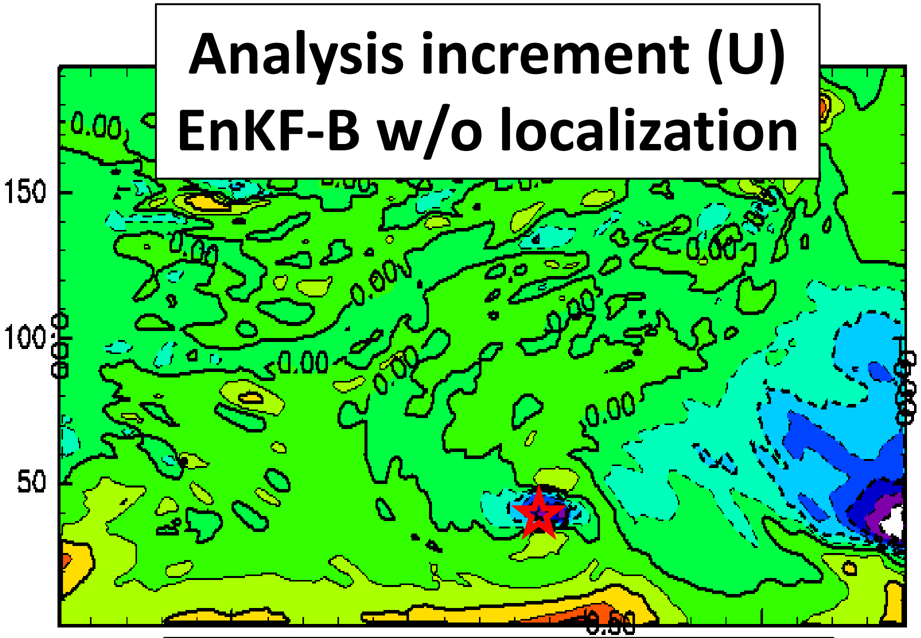

Irrelevant to the vortex structure

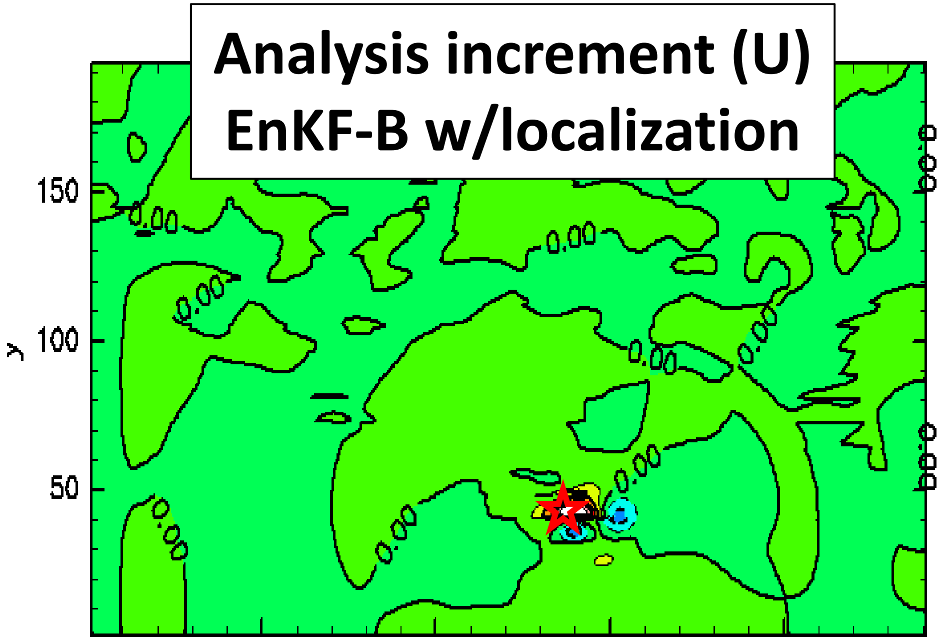
EnKF-based **B** (Hybrid EnKF-4DVAR)

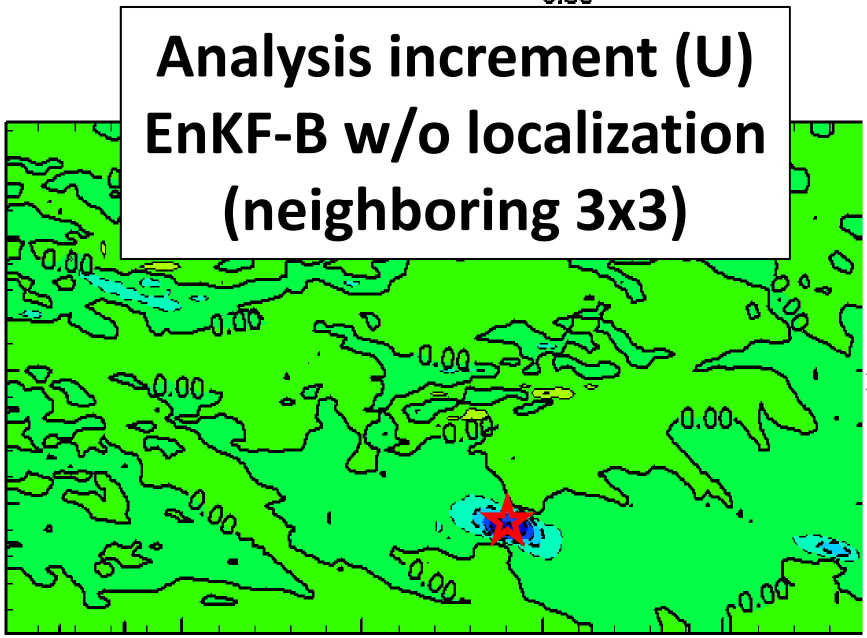


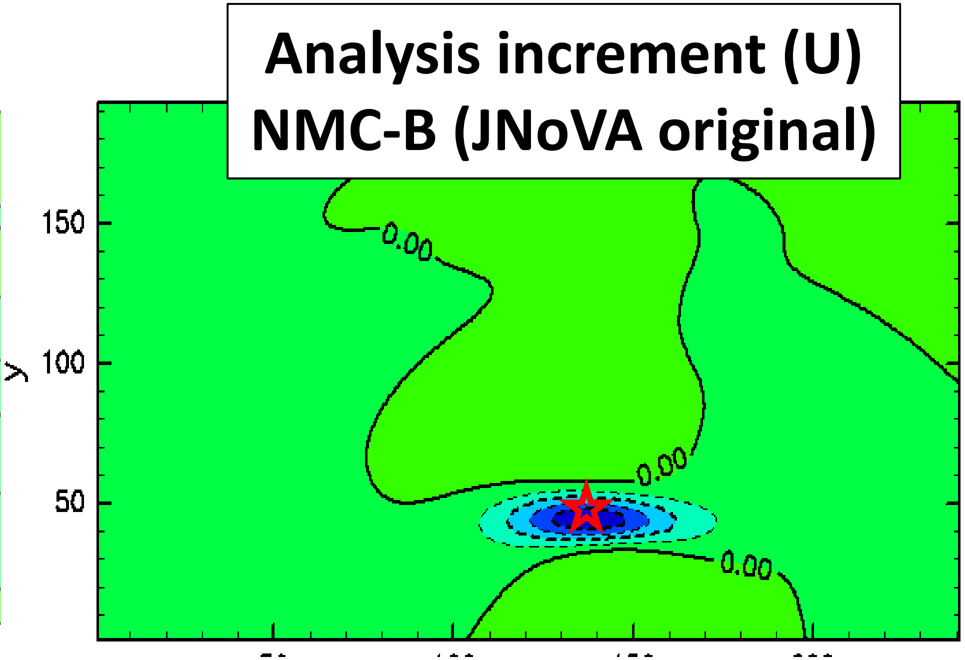

Consistent with vortex displacement

# JNoVA: 1-obs. DA around typhoon Talas



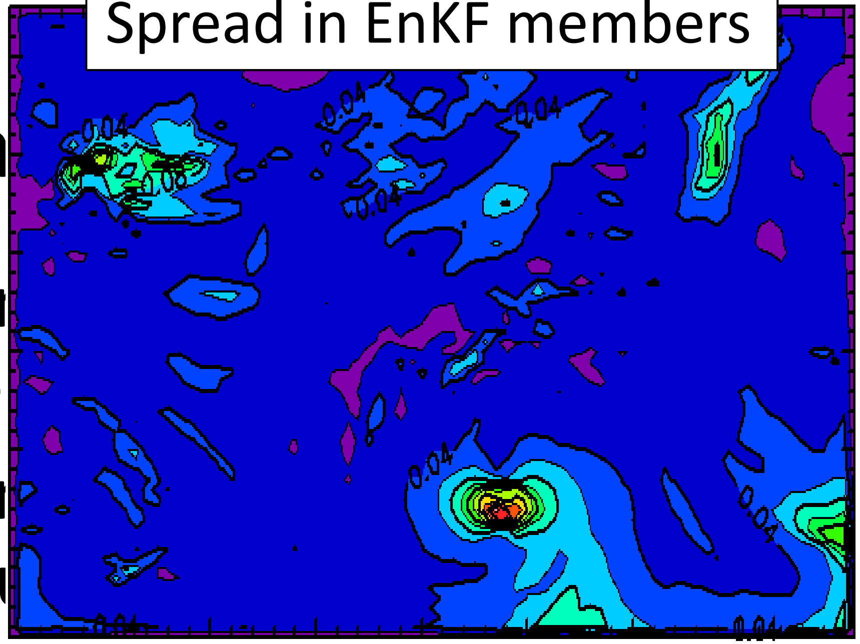



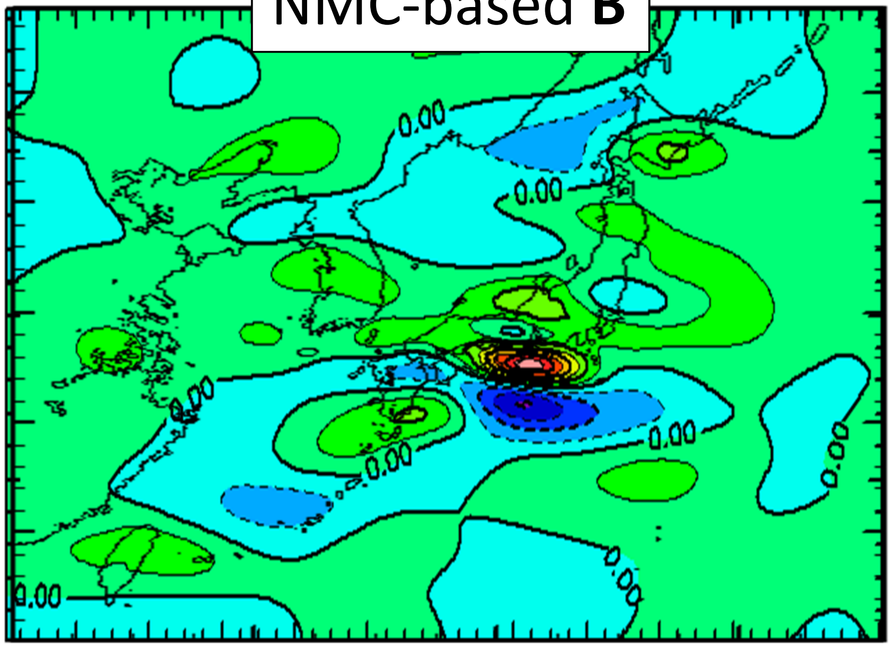
# Analysis increment at initi
# (For real data durin

- Analysis increment by usi
located in the dense obse
- Analysis increment by usi
around TC and frontal clo

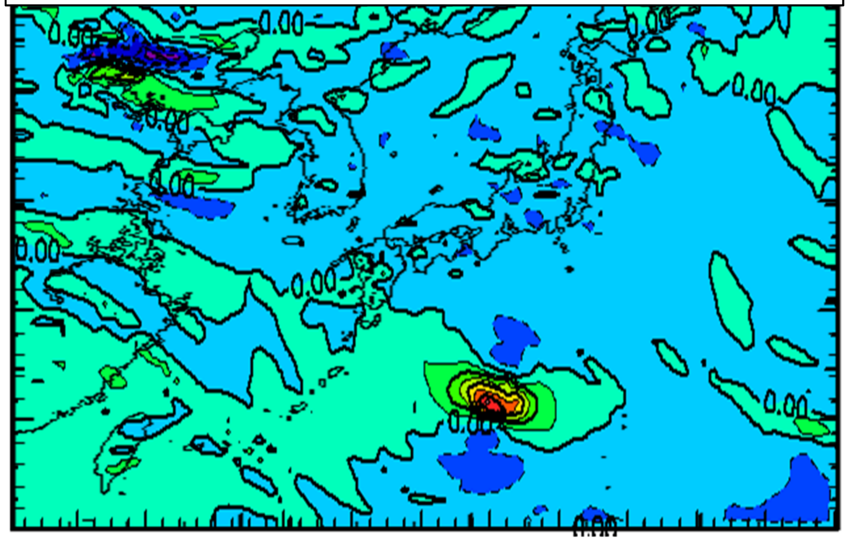
Spread in EnKF members




NMC-based **B**




EnKF-based **B** w/o localization
(neighboring 3x3)



# Summary

- **Solution of BLUE:**
  Singular vector of $\mathbf{R}^{-1/2}\mathbf{HMB}^{1/2}$ with large singular value determines the structure of analysis increment.
- **Hybrid EnKF-4DVAR**

  EnKF-based flow-dependent **B** is used instead of NMC-based climatological **B.**
- **Implementation to JNoVA**
  Currently ongoing. Preliminary experiments seem to work well. Further tests for localization and other tasks are needed.